

ELICITATION RECORD – Part 3 – Multivariate

Copula Distribution

Elicitation title	As in the Part 1 form
Session	As in the Part 1 form
Date	As in the Part 1 form
Quantity	<p>The uncertain set of quantities whose joint distribution is to be elicited.</p> <p>[This SHELF 3 (Multivariate) form is for elicitation of a distribution for a set of uncertain quantities. A flexible family of distributions for fitting multivariate distributions with arbitrary marginal distributions is the multivariate normal copula family. See the document “Multivariate Elicitation” for advice on the use of this form.]</p>
Anonymity	Record here the codes that will be used to identify experts in this template. For instance, “In this record, experts are identified by letters A, B, C and the facilitator by Z.”
Start time	Time when this part of the elicitation started

Definition	<p>Repeat the definitions of these quantities from Part 1. Give them symbols to facilitate the recording of judgements about them.</p> <p>They will be called X_1, X_2, \dots, X_k in these notes.</p>
Marginal distributions	<p>Using your choice of method, complete a SHELF 2 form for each of the quantities X_i separately.</p> <p>Record here these elicited marginal distributions and their medians. The median for X_i will be called m_i in these notes.</p>
Quadrant probabilities	<p>Elicit and record here the concordance probabilities</p> $P_{ij} = \Pr([X_i > m_i \text{ AND } X_j > m_j] \text{ OR } [X_i < m_i \text{ AND } X_j < m_j])$ <p>for each $i < j$.</p> <p>[For large k this will be impractical; just for $k = 6$ quantities, it will already be necessary to elicit 15 concordance probabilities.</p> <p>Refer to the slide set “Concordance Probabilities” for ways to explain to the experts what is required, and to help them to challenge and refine their judgements. The experienced facilitator may prefer to present these ideas in their own way, but otherwise it is recommended that the slide set should be used directly as a presentation to the experts.</p> <p>The facilitator may choose to use the SHELF 2 approach of</p>

	having individual judgements followed by discussion and group judgements, but may alternatively choose just to elicit group “consensus” judgements.]
Fitting	<p>Fitting the multivariate normal copula distribution requires a rank correlation matrix R to be chosen that best fits the elicited concordance probabilities.</p> <p>[First convert the concordance probabilities to correlations by $r_{ij} = \sin(\pi P_{ij} - \pi/2)$ Together with $r_{ii} = 1$ and $r_{ji} = r_{ij}$, these form the elicited rank correlation matrix R.</p> <p>However, R must be non-negative definite, and it is possible that the elicited R does not satisfy this condition. (This is more likely for larger k.) In that case, the facilitator should discuss with the experts how they might modify their concordance judgements. The “SHELF” software provides some help with doing this, by identifying in suitable cases a range of alternative values for an individual concordance probability that would fit with the other elicited values.]</p>
Feedback	<p>Present feedback to the expert on the implications of the fitted copula distribution.</p> <p>[If the fitted R is different from the elicited R, feedback should include the concordance probabilities implied by the fitted R. The “SHELF” software includes functions to compute these and other feedback values.]</p>
Chosen distribution	Record the finally agreed copula distribution, in the form of marginal distributions and rank correlation matrix.
Discussion	The facilitator should now record any difficulties that arose during the elicitation. Also the experts' reactions to the process and to the final fitted distribution.

End time	Time when elicitation of this distribution was completed.
Attachments	List any attachments, which should include all the SHELF 2 forms.