

ELICITATION RECORD – Part 3

Multivariate Dirichlet Distribution

Elicitation title	As in the Part 1 form
Session	As in the Part 1 form
Date	As in the Part 1 form
Quantity	<p>The set of proportions whose distribution is to be elicited.</p> <p>[This SHELF 3 form is for elicitation of a distribution for a set of proportions that sum to 1. The most convenient distributions in this case are the Dirichlet family of distributions, and this form follows a procedure to fit a Dirichlet distribution to experts' knowledge about a set of proportions. If it is found that the experts' beliefs are not adequately represented by a Dirichlet distribution, this procedure must be abandoned. See the document "Multivariate Elicitation" for advice on the use of this form.]</p>
Anonymity	Record here the codes that will be used to identify experts in this template. For instance, "In this record, experts are identified by letters A, B, C and the facilitator by Z."
Start time	Time when this part of the elicitation started

Definition	Repeat the definition of this quantity from Part 1. Give it a symbol to facilitate the recording of judgements about it. It will be called $\mathbf{X} = (X_1, X_2, \dots, X_k)$ in these notes.
Elicit beta distributions	<p>Using your choice of method, complete a SHELF 2 form for each of the proportions X_i separately. The fitted distribution in each case should be a beta distribution.</p> <p>[If for any X_i the experts feel that a beta distribution does not represent their beliefs about X_i, then this procedure for eliciting a Dirichlet distribution fails. In that case, this record should be terminated. The facilitator may then adopt another method to elicit the experts' beliefs about \mathbf{X}, but there is as yet no SHELF procedure to deal with a set of proportions when a Dirichlet distribution is not suitable.]</p> <p>Record here the parameters of the elicited beta distributions. We will refer to the parameters of the distribution of X_i as d_i and e_i,</p>
Check and adjust means	Having ascertained that individual beta distributions represent the knowledge of the experts, we need to check consistency of the beta distributions with the fact that the proportions must sum to 1.

	<p>Compute mean values $m_i = d_i / (d_i + e_i)$ and their sum $M = m_1 + m_2 + \dots + m_k$. Then adjust the fitted beta distributions so that their means sum to 1: replace d_i with $d_i^* = d_i/M$ and e_i with $e_i^* = d_i + e_i - d_i^*$.</p> <p>[If M is far from 1, then this indicates possible misunderstanding by the experts. This should be discussed with the experts before simply applying the above adjustment.]</p>
Fitting a Dirichlet to the elicited beta distributions	<p>Letting $n_i^* = d_i^* + e_i^*$, the Dirichlet distribution requires that all of these equal a common value n. The next step is therefore to find a compromise value of n that fits the elicited beta distributions as well as possible.</p> <p>[Various possible choices of n can be considered. In general, it should lie between the minimum and the maximum of the individual n_i^* values. Two natural choices are the mean and the median of the n_i^*s. The SHELF software allows for a slightly more sophisticated choice that optimises the fit of the Dirichlet to the standard deviations of the elicited beta distributions.]</p> <p>Having chosen n, the parameters of the Dirichlet distribution are given by $\mathbf{p} = (p_1, p_2, \dots, p_k)$, where $p_i = n \cdot d_i^* / n_i^*$.</p>
Feedback	<p>Present feedback to the expert on the implications of the fitted Dirichlet distribution. Ideally, this would include showing the probabilities or quantiles elicited from the experts for each X_i compared with those implied by the fitted Dirichlet.</p> <p>[In general, the Dirichlet implies a little more information about each X_i whose $n_i^* < n$, and a little less information about X_is whose $n_i^* > n$. The experts are being asked to accept these compromise values. If the original n_i^*s were close together, the compromise will be minor, so the experts are likely to find the fitted values acceptable. Otherwise, it may be necessary to try an alternative n. Ultimately, if no compromise value of n is acceptable, then this procedure for eliciting a Dirichlet distribution fails. In that case, this record should be terminated. The facilitator may then adopt another method to elicit the experts' beliefs about \mathbf{X}, but there is as yet no SHELF procedure to deal with a set of proportions when a Dirichlet distribution is not suitable.]</p>
Chosen distribution	Record the finally agreed Dirichlet distribution.
Discussion	The facilitator should now record any difficulties that arose during the elicitation of the Dirichlet distribution, also the experts' reactions to the process and to the final fitted distribution.

End time	Time when elicitation of this distribution was completed.
Attachments	List any attachments, which should include all the SHELF 2 forms.